

ASPECTS OF DUALITIES

AMIT GIVEON

Theory Division, CERN, CH-1211, Geneva 23, Switzerland

MASSIMO PORRATI

*Dept. of Physics, NYU, 4 Washington Pl., New York NY 10003, USA
and
Rockefeller Univ., New York NY 10021-6399, USA*

In this talk, some aspects of duality symmetries are presented.

In the last few years, duality symmetries in field theory and in string theory have been studied extensively. Let us list some of them:

- *Electric-Magnetic Dualities:* strong-weak coupling duality which relates apparently different field theories under the interchange of electric degrees of freedom with magnetic degrees of freedom (for a review, see ^{1,2}).
- *T-Duality:* target space duality in string theory relates different string backgrounds which are physically equivalent (for a review, see ³).
- *S-Duality:* strong-weak coupling duality in string theory (for a review, see ⁴).
- *U-Duality:* intertwines T-duality and S-duality ⁵.
- *String-String Dualities:* relate different string theories to each other (for a review, see ⁶).
- *World-Sheet \leftrightarrow Target-Space Dualities:* relate a theory on a world-sheet and target-space (Σ, T) to a theory on a different world-sheet and a different target-space $(\tilde{\Sigma}, \tilde{T})$ ⁷.

The new results in this talk^a are based on work in ref. ⁸. We present duality symmetries in 4D, Abelian gauge theories which involve also the (Euclidean, compact) space-time M^4 . Such dualities

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– rather mysterious from the 4D point of view – are better understood as geometrical symmetries of theories in higher dimensions, compactified to M^4 on some internal space.

Explicitly, we find dualities which relate a pair (M^4, τ) to a different pair $(\tilde{M}^4, \tilde{\tau})$, where τ is the complex coupling-constant matrix of a $U(1)^r$ gauge theory on M^4 , and $\tilde{\tau}$ is the dual coupling-constant matrix of a $U(1)^{\tilde{r}}$ gauge theory on \tilde{M}^4 .

Some of the dualities considered can be understood as the consequence of string dualities, in the limit where gravity is decoupled. A simple example is the string-string triality which relates the heterotic string compactified on $T^4 \times T^2$ to Type II strings on $K^3 \times T^2$. In the low-energy effective field theory, at the limit $M_{Planck} \rightarrow \infty$, this triality becomes part of the electric-magnetic duality group of an $N = 4$, $U(1)^4$ supersymmetric Yang-Mills (YM) theory on $\mathbf{R}^{3,1}$.

In the rest of the talk, we continue by touring more examples which involve also space-time:

1. *S \leftrightarrow U Duality in $N = 4$ YM Theory and Self-Dual Superstring in 6D:* consider $SU(2)$, $N = 4$ YM theory on $M^4 = S^1_\beta \times S^1_R \times T^2_U$, where β is the inverse temperature, R is the radius of a circle, and T^2_U is a 2-torus with complex structure U . Let S denote the complex gauge coupling. The partition function at large scalar VEVs was computed in ⁹. It turns out that not only is it invariant under $SL(2, \mathbf{Z})$, S -duality and the geometrical $SL(2, \mathbf{Z})$ transformations acting on the complex structure, but also under the transfor-

mation interchanging S with U . This $S \leftrightarrow U$ duality is a mysterious symmetry from the $4D$ gauge theory point of view. As explained in⁸, we can understand the origin of this duality from the manifest geometrical symmetry of a compactified self-dual superstring in $6D$. Explicitly, by reducing a theory of a self-dual 2-form in $6D$ to $2D$ on $T_U^2 \times T_S^2$, we obtain the same partition function as above. The $S \leftrightarrow U$ duality is a consequence of the manifest $T_U^2 \leftrightarrow T_S^2$ geometrical symmetry. The technical details are given in⁸.

2. *Compactification of a 2-Form Theory in 6D and Triality*: by compactifying a theory of a self-dual 2-form in $6D$ on $T_T^2 \times T_U^2 \times T_S^2$, we obtain a triality symmetry under the permutations of S, T and U . In a certain large volume limit, the partition function is identical to the classical part of the one-loop partition function of a $2D$ sigma-model with a T^2 target-space. In string theory, this triality – observed sometime ago¹⁰ – is rather mysterious, because T is the complex structure of the world-sheet torus, while U and S are the complex structure and the Kähler structure of the target-space torus, respectively. However, for the 2-form theory on $T_T^2 \times T_U^2 \times T_S^2$, this triality is the geometrical symmetry permuting the three 2-tori.

3. *Compactifications of a 2-Form Theory in 6D and More Dualities*: a generalization of the previous example is to compactify a self-dual 2-form theory in $6D$ on $\Sigma_g \times T_S^2 \times \tilde{\Sigma}_{\tilde{g}}$, where Σ_g and $\tilde{\Sigma}_{\tilde{g}}$ are (different) Riemann surfaces with genus g and \tilde{g} , respectively. The partition function is identical to the classical partition function of a $2D$ sigma-model with world-sheet Σ_g (alternatively, $\tilde{\Sigma}_{\tilde{g}}$) and target-space $T^{2\tilde{g}}$ (alternatively, T^{2g}) whose metric and antisymmetric background are parametrized by S and the period matrix of $\tilde{\Sigma}_{\tilde{g}}$ (alternatively, by the period matrix of Σ_g). We find a world-sheet \leftrightarrow target-space duality between the pairs

$$\{\Sigma_g, T^{2\tilde{g}}(\tilde{\Sigma}_{\tilde{g}})\} \leftrightarrow \{\tilde{\Sigma}_{\tilde{g}}, T^{2g}(\Sigma_g)\}.$$

This duality is a manifest consequence of the geometrical symmetry interchanging $\Sigma_g \leftrightarrow \tilde{\Sigma}_{\tilde{g}}$ in the compactified 2-form theory. Similar world-sheet \leftrightarrow target-space dualities

were observed in⁷. These can be understood as the $\Sigma_g \leftrightarrow \tilde{\Sigma}_{\tilde{g}}$ geometrical symmetries of a compactified $6D$ 2-form theory, which instead of self-duality obey other conditions⁸.

4. $(M^4, \tau) \leftrightarrow (\tilde{M}^4, \tilde{\tau})$ *Duality and Compactifications of a Self-Dual 4-Form Theory in 10D*: a reduction of a $10D$ self-dual 4-form theory to $4D$ on $M^4 \times T_S^2$ gives a $4D$, $U(1)^{b_2}$ gauge theory, where M^4 is a 4-manifold with $b_1(M^4) = 0$; b_1, b_2 are the Betti numbers of M^4 . Upon further compactification on $M^4 \times T_S^2 \times \tilde{M}^4$ we find a manifest symmetry under the interchange $M^4 \leftrightarrow \tilde{M}^4$. This translates into a rather “puzzling” duality of the $4D$ gauge theory:

$$\{M^4, \tau(\tilde{M}^4)\} \leftrightarrow \{\tilde{M}^4, \tilde{\tau}(M^4)\},$$

where $\tau(\tilde{M}^4)$ is the gauge-coupling matrix given in terms of the geometrical data of \tilde{M}^4 , and $\tilde{\tau}(M^4)$ is the dual gauge-coupling matrix given in terms of the geometrical data of M^4 . This duality relates a theory on space-time manifold M^4 and coupling-constant matrix τ to a theory with a different 4-manifold and a different gauge-coupling matrix. This construction sometimes works for more than the partition function of an Abelian gauge theory. For $M^4 = K^3$ and $\tilde{M}^4 = \tilde{K}^3$, this construction can be embedded in an $N = 4$ compactification of type IIB string on $K^3 \times T_S^2 \times \tilde{K}^3$.

To summarize:

- Duality groups of Abelian gauge theories on 4-manifolds and their reduction to $2D$ were considered.
- The duality groups include elements that relate different space-times in addition to relating different gauge-coupling matrices.
- We interpreted such dualities as geometrical symmetries of compactified theories in higher dimensions.
- In particular, we considered compactifications of a (self-dual) 2-form in $6D$, and of a self-dual 4-form in $10D$.
- Relations with a (conjectured) self-dual superstring in $6D$ and with Type IIB superstring were mentioned.

- There are many more duality symmetries of the classical partition sum of free $U(1)^r$ gauge theories on M^4 which were not discussed here.
- To recover all symmetries of $4D$ gauge theories, and their possible geometrical origin from higher dimensions, is a problem which may shed more light on the non-perturbative dynamics of gauge theories and strings.
- Here we considered aspects of this problem in some simple, yet probably significant cases.

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